

# Quantifying the accuracy of a radial approximation for pile heat exchangers.

Nick Woodman<sup>1</sup>, Fleur Loveridge<sup>2</sup>, Saqib  
Javed<sup>3</sup>, Johann Claesson<sup>3</sup>.

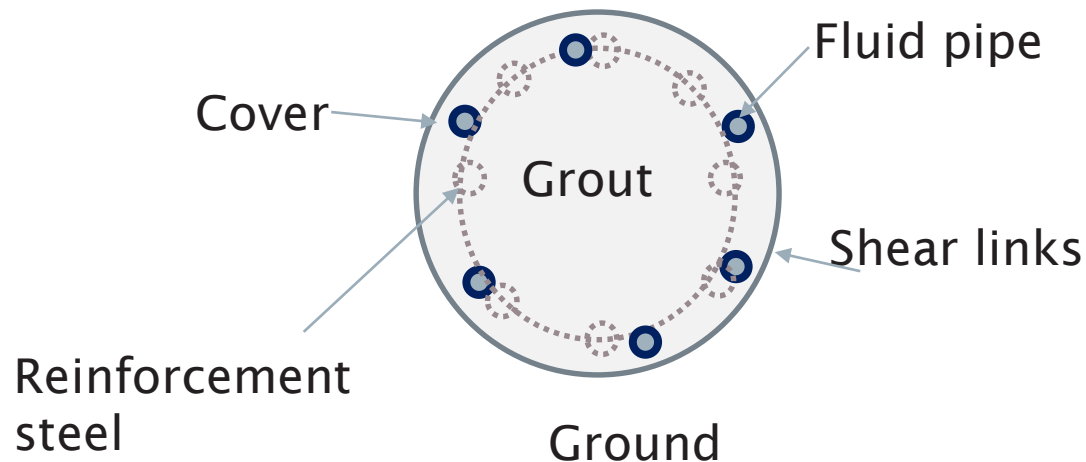
1. University of Southampton, 2. University of Leeds, 3. Lund University

# Introduction

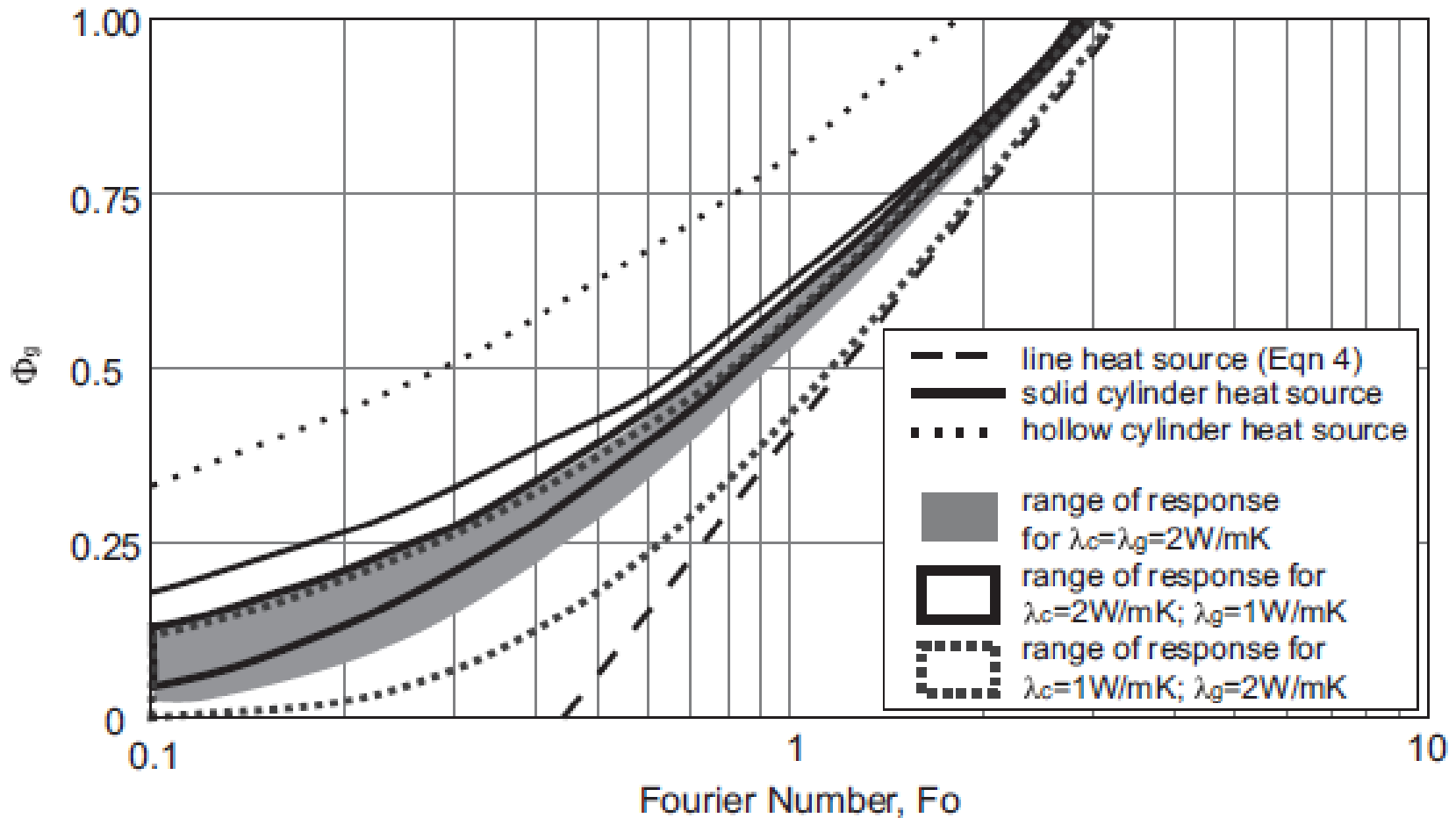
## How to model thermal piles with sufficient accuracy?

...using simplest possible acceptable model ('fit for purpose')

...taking into account location of pipes, steel, diffusivity contrast between pile grout and ground?



# Pile responses



*Loveridge & Powrie, 2013*

# “Any arrangement”....

DFDP 2 being drilled 2014 Whataroa Valley, Southern Alps, New Zealand  
‘Terminated’ at 818 m. . .  
Groundwater Heads reached 60m  
Temperatures reached 110°C

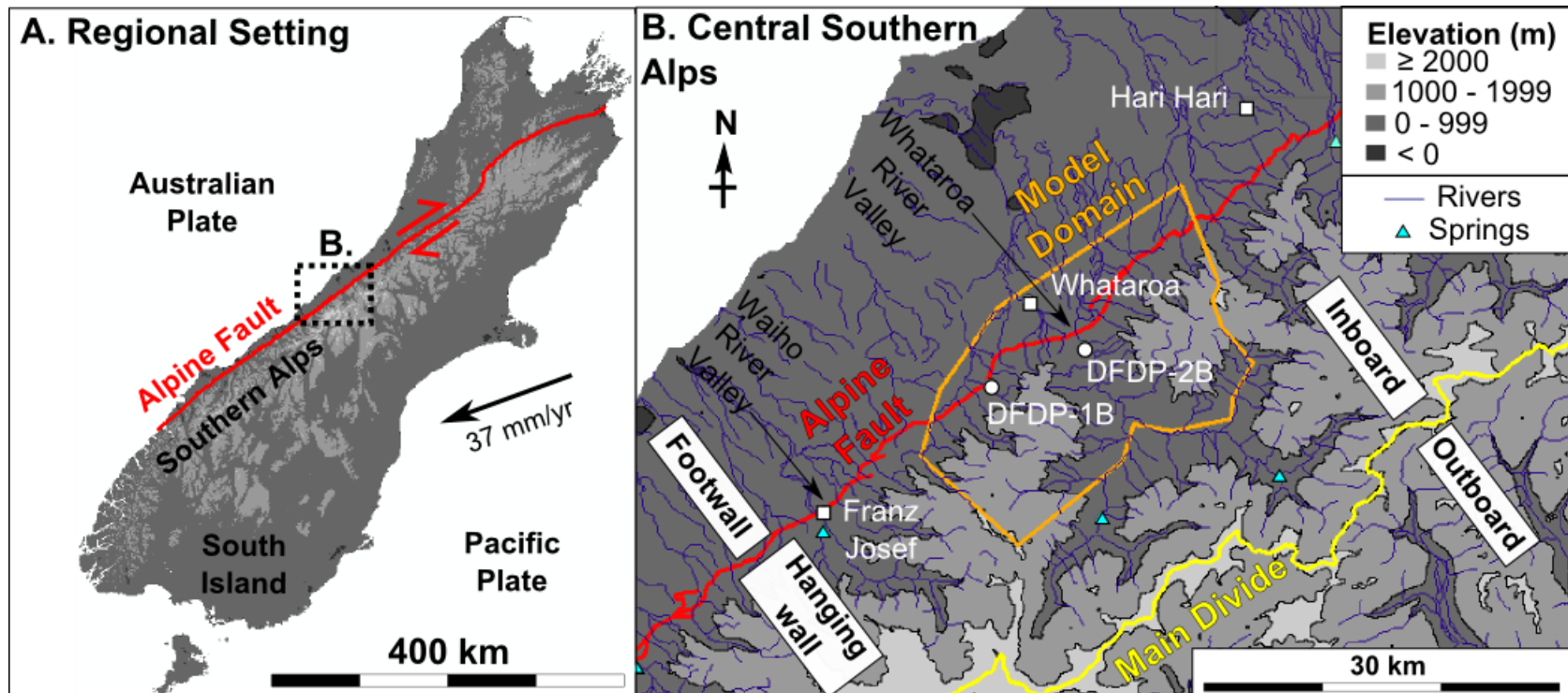


“Any arrangement” ....

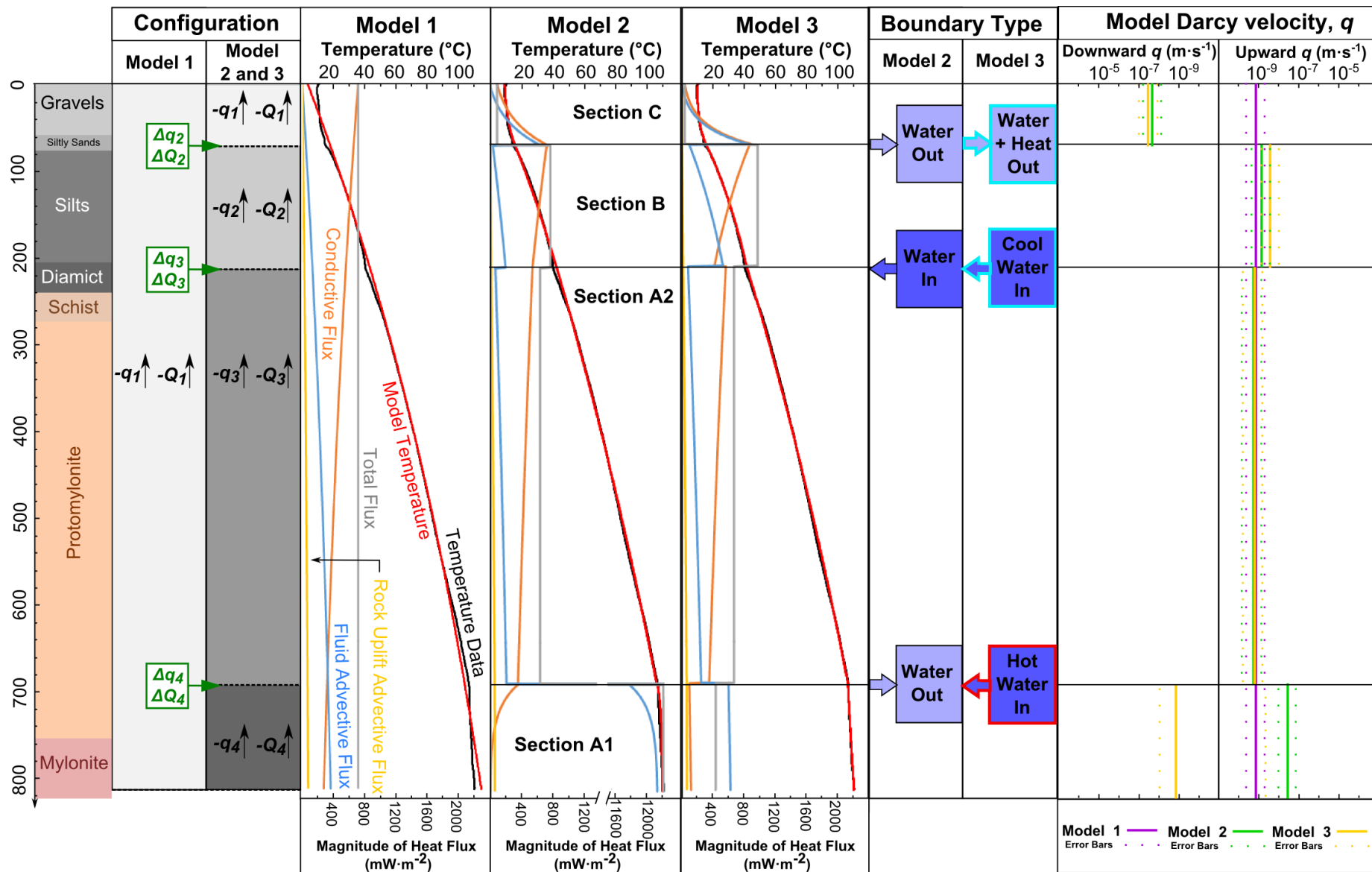
# LETTER

doi:10.1038/nature22355

## Extreme hydrothermal conditions at an active plate-bounding fault

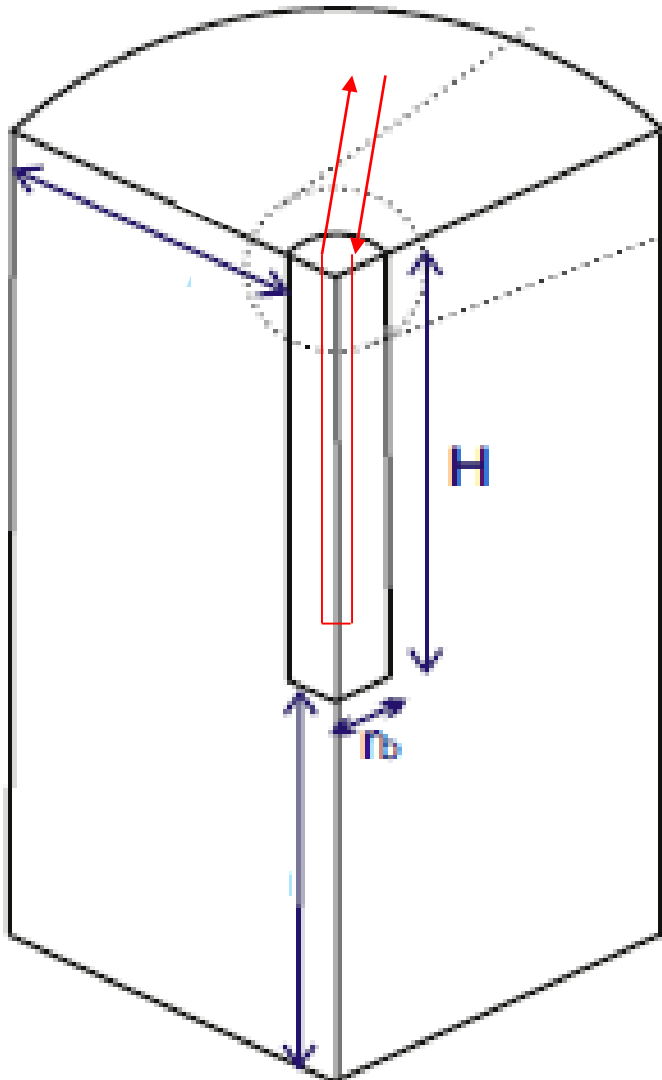


# 1D insights



# Pile Geometry

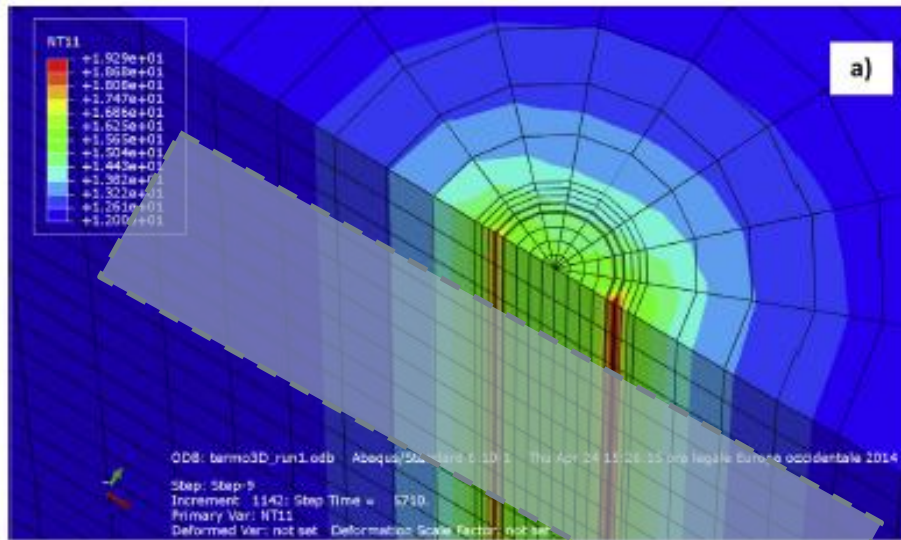
Aspect Ratio,  $AR=H/(2r_b)$



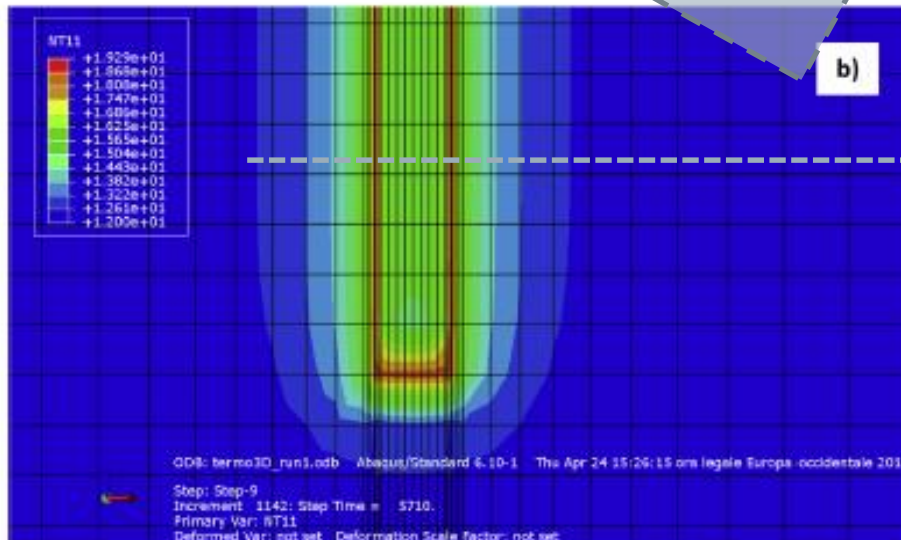
Hemmingway, Phil; Long, Michael (Michael M.)

# Explicit 3D

F. Cecinato, F.A. Loveridge / Energy 82 (2015) 1021–1033



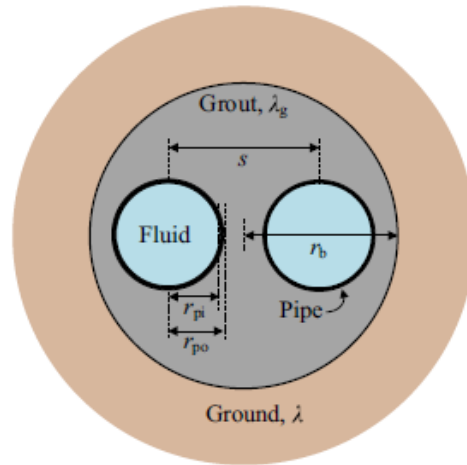
2D cut



2D cut

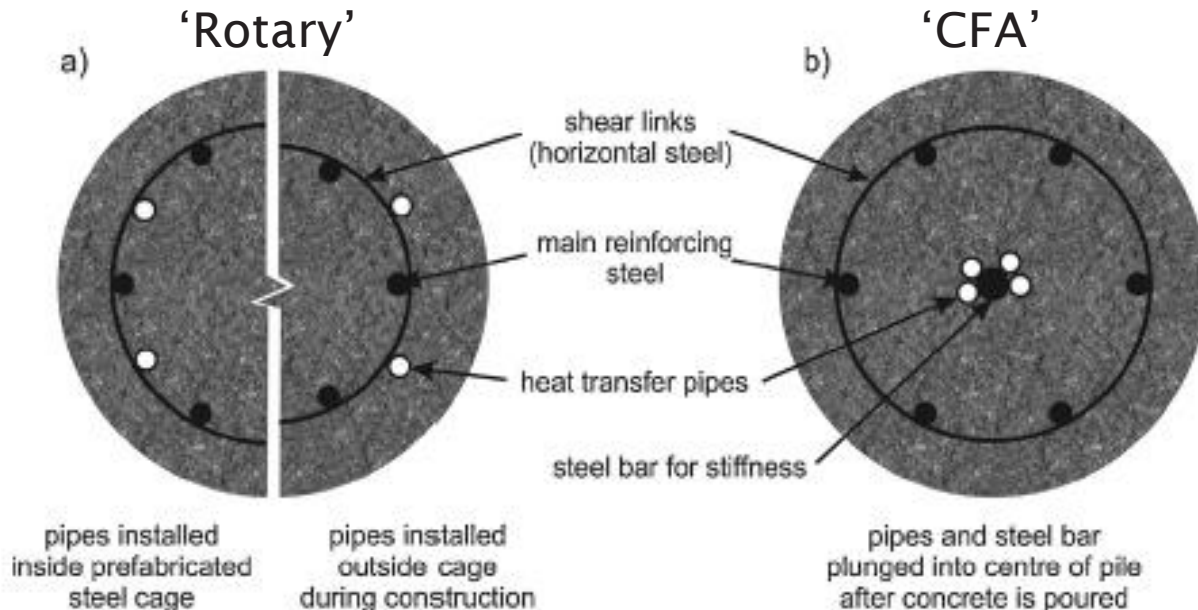


# Borehole Heat Exchanger



- $r_b \sim 0.1 \text{ m}$
- $H \sim 10\text{-}100 \text{ m}$
- $AR \sim 100$  (*neglect axial heat-flow*)
- Relatively small thermal mass of grout (*assume steady-state resistance for grout*)

# Pile Heat Exchanger



- $r_b \sim 1 \text{ m}$
- $AR \sim 10$  (*axial heat-flow*)
- Significant thermal mass (*non-steady-state temperatures in grout except at late-time*)
- More U-tubes

# 2D model assumptions

Well-mixed (isothermal) fluid, constant uniform power

Neglect convective resistance (easily added)

Steady-state pipe resistance

2D-axially symmetric heat flow:

- stationary groundwater
- homogeneous
- initial steady-state
- neglect axial heat flow

No other interactions or constitutive relationships

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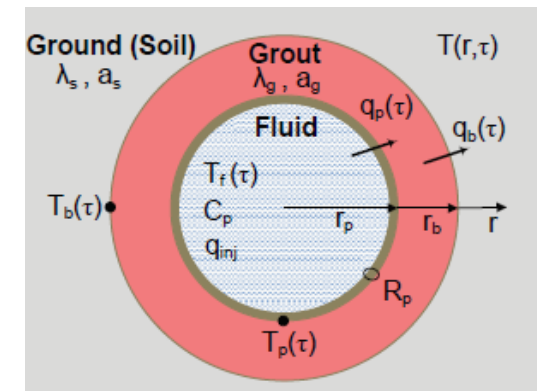
LV-11-C001

# New Analytical and Numerical Solutions for the Short-term Analysis of Vertical Ground Heat Exchangers

Saqib Javed, P.E.

*Student Member ASHRAE*

Johan Claesson, Ph.D.

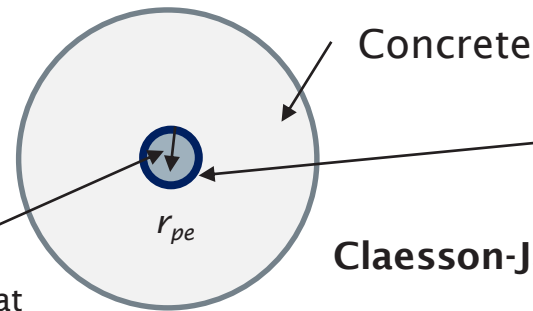
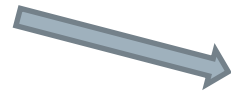
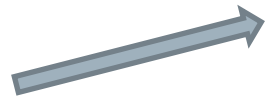
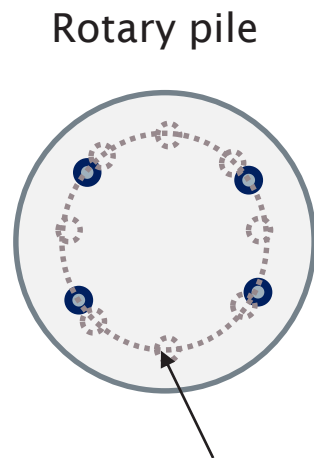


*Call this "Claesson-Javed Radial Model - CJRM"*

# Model equivalence (rotary)

Explicit Geometry (EGM)  
*Numerical model*

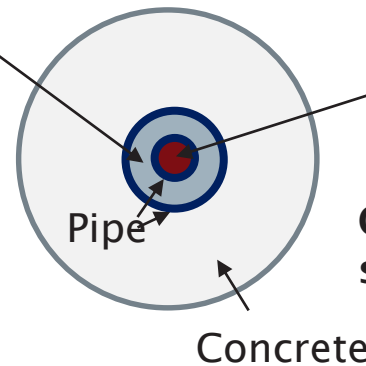
Equivalent Radial Geometry  
*Semi-analytical model*



Pipe modelled as steady-state resistance,  
 $R_{pe} = R_p / 4$

Total fluid heat capacity kept constant

Isothermal store,  
Heat capacity varied to capture 'enclosed' part of pile

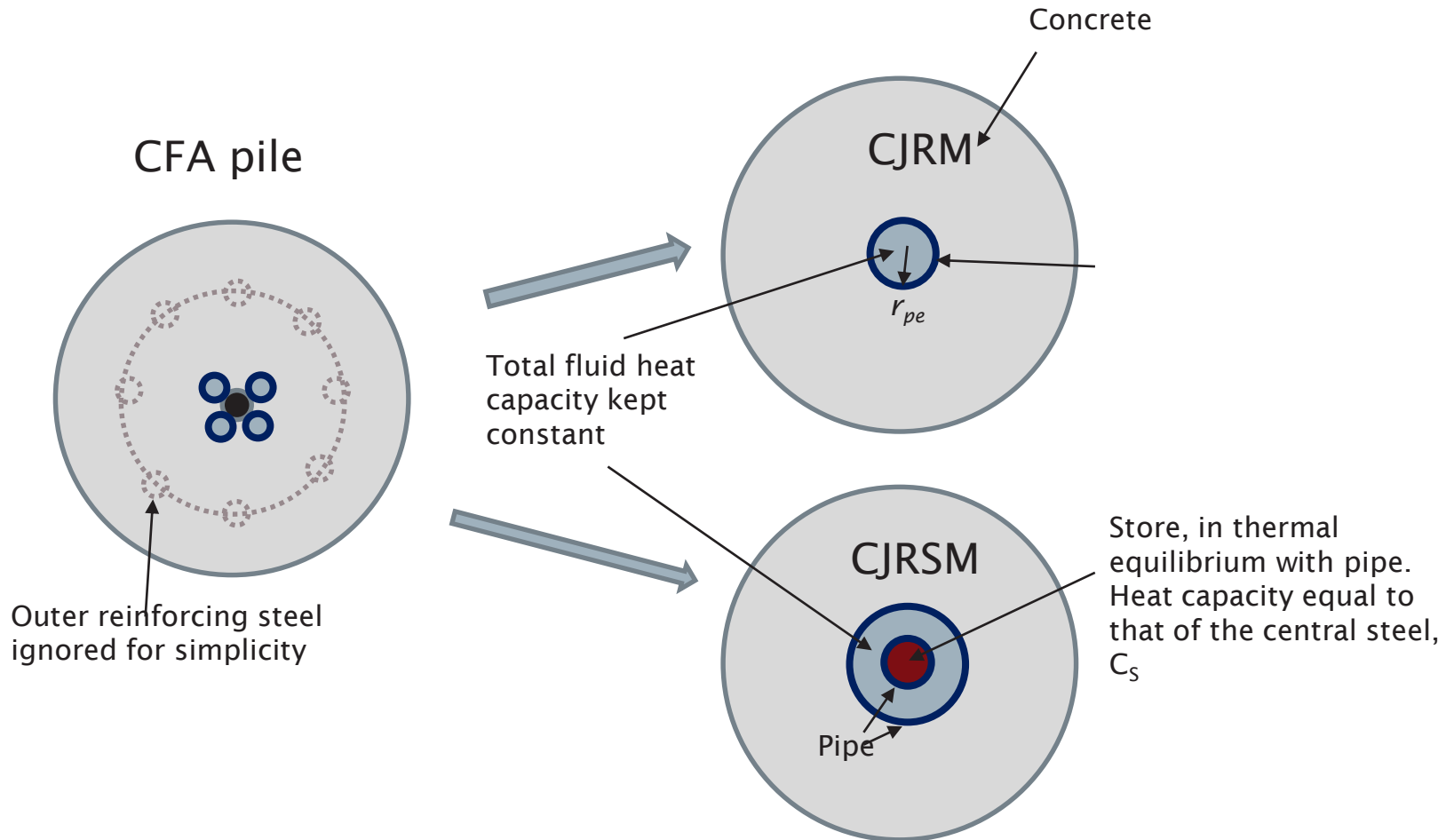


Keep the ground and concrete properties the same

# Model equivalence (CFA)

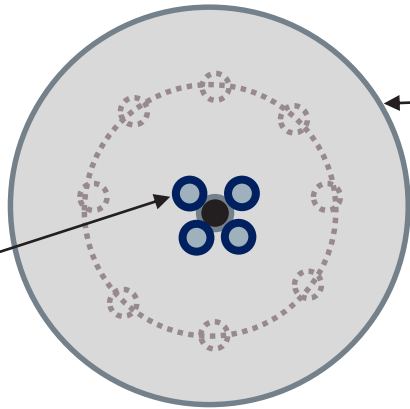
Explicit Geometry (EGM)

Equivalent Radial Geometry

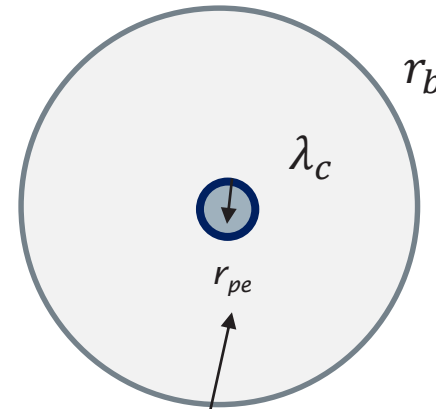


# Borehole resistance (steady-state)

EGM

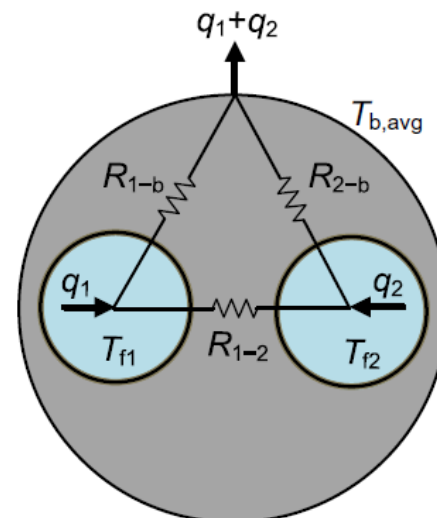
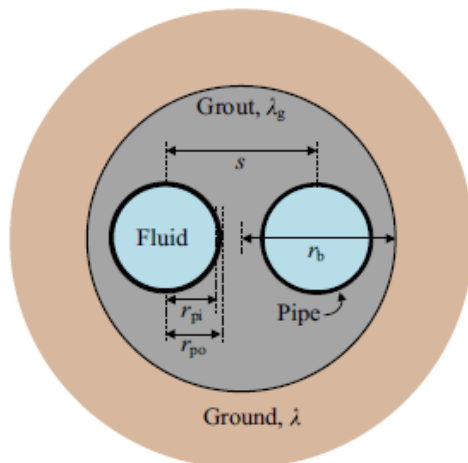


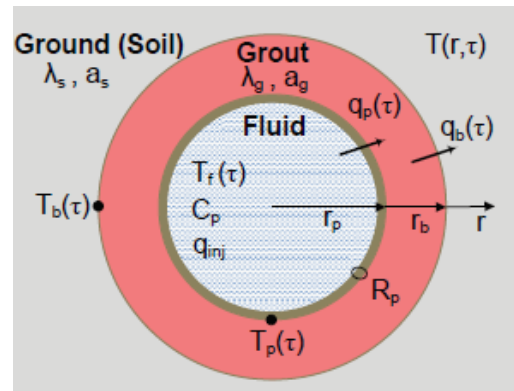
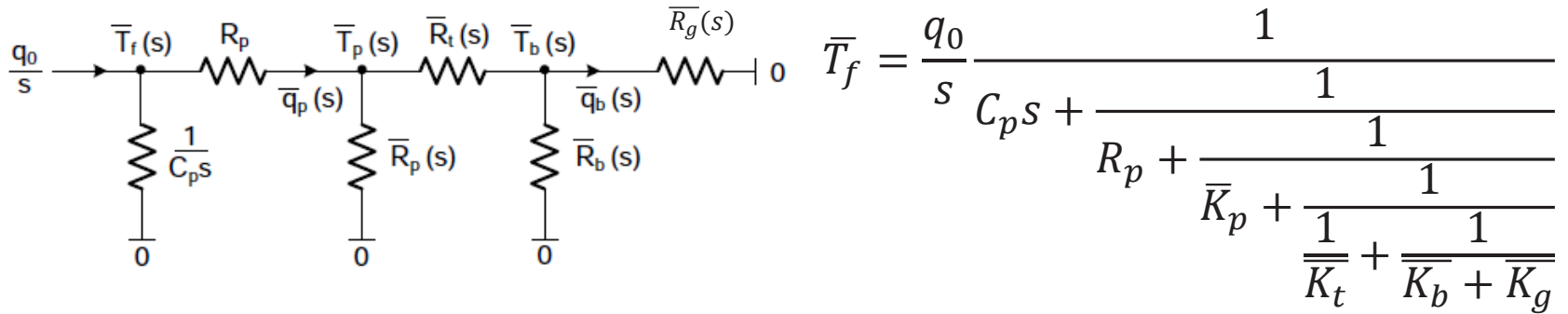
CJRM



$$\bar{T}_{po} - \bar{T}_b = qR_b = \frac{q}{2\pi\lambda_c} \ln\left(\frac{r_b}{r_{pe}}\right)$$

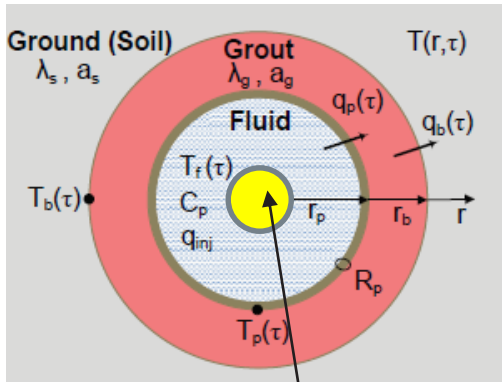
Javed & Spitler (2017) 10 methods vs. 10<sup>th</sup> order multi-pole.



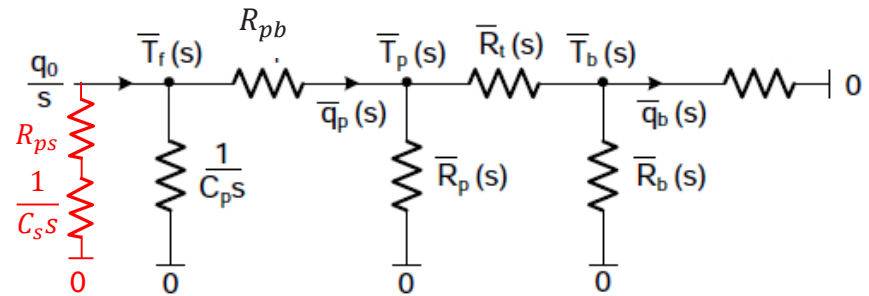


$$\bar{K}_g = \frac{1}{R_g} = 2\pi\lambda \frac{\tau_g u [J_1(\tau_g u) - iY_1(\tau_g u)]}{J_0(\tau_g u) - iY_0(\tau_g u)} \quad \text{etc}$$

$$\bar{T}_f = \frac{q_0}{s} \frac{1}{C_p s + \frac{1}{R_{ps} + \frac{1}{C_s s}} + \frac{1}{R_{pb} + \frac{1}{\bar{K}_p + \frac{1}{\frac{1}{\bar{K}_t} + \frac{1}{\bar{K}_b + \bar{K}_g}}}}$$



$C_s$





## A Generalized Radial Flow Model for Hydraulic Tests in Fractured Rock

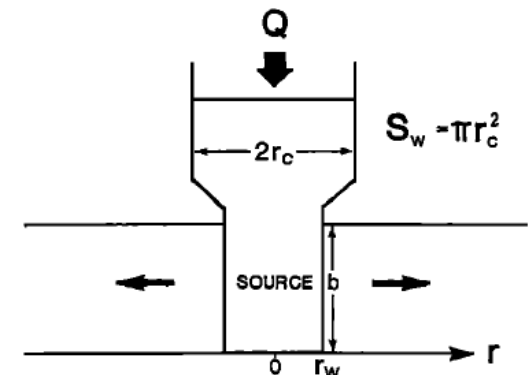
J. A. BARKER

*British Geological Survey, Wallingford, Oxfordshire, United Kingdom*

$$\bar{Q}(p)/\bar{H}(p) = pS_w + K_f b^{3-n} \alpha_n r_w^{n-2} \Phi_v(\mu) / [1 + s_f \Phi_v(\mu)] \quad (21)$$

where the function  $\Phi_v(z)$  is defined by

$$\Phi_v(z) = zK_{v-1}(z)/K_v(z)$$



## Modelling doublets and double porosity

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**Abstract:** A simple model has been developed as a scoping tool for transport between an injection well and an abstraction well pumping at the same rate (i.e. a doublet) in a fractured porous rock. This model is aimed primarily at the planning and preliminary interpretation of tracer tests and trial heat exchange using thermal doublets in the Chalk aquifer. The model is essentially a particular case of transport along

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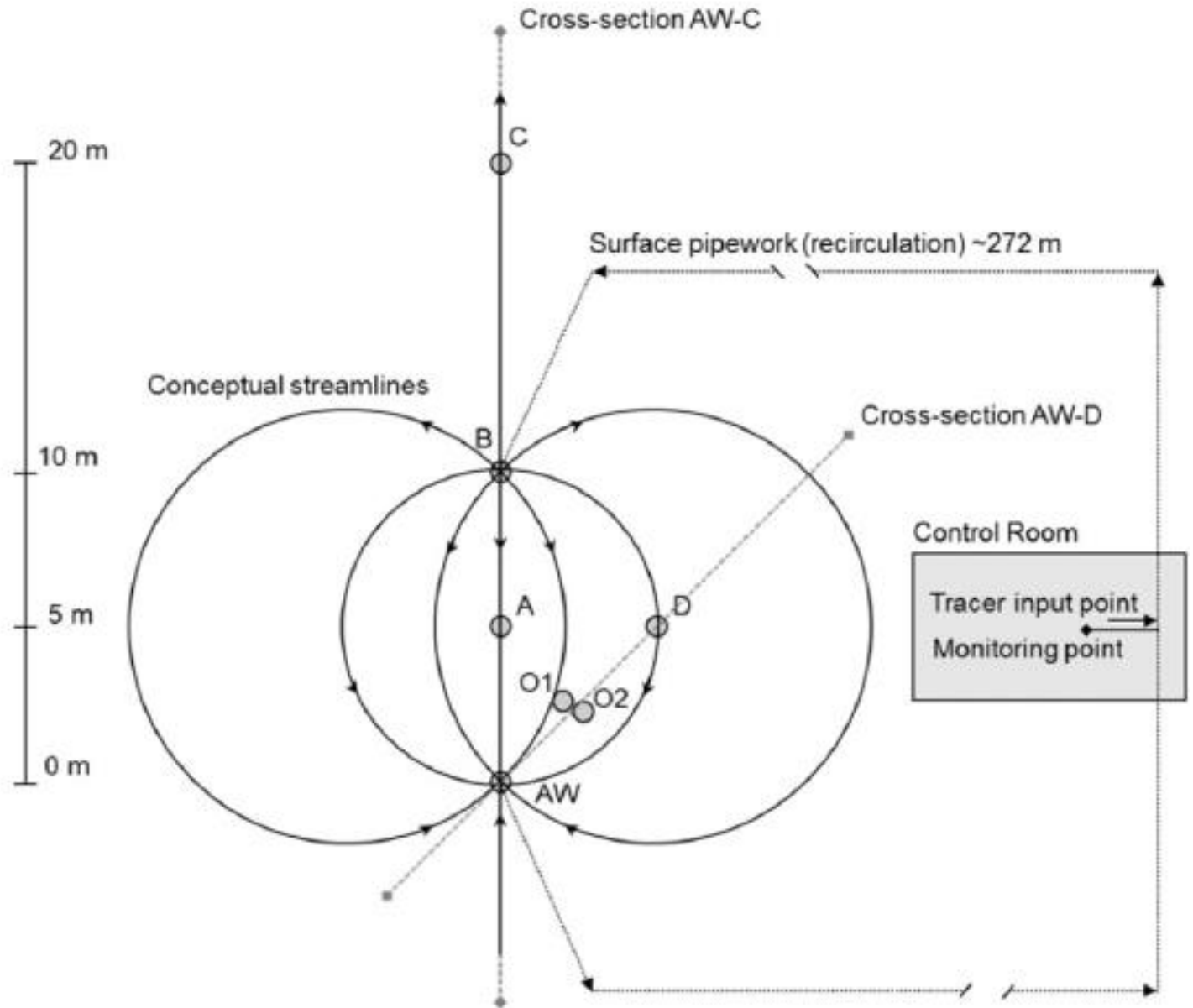


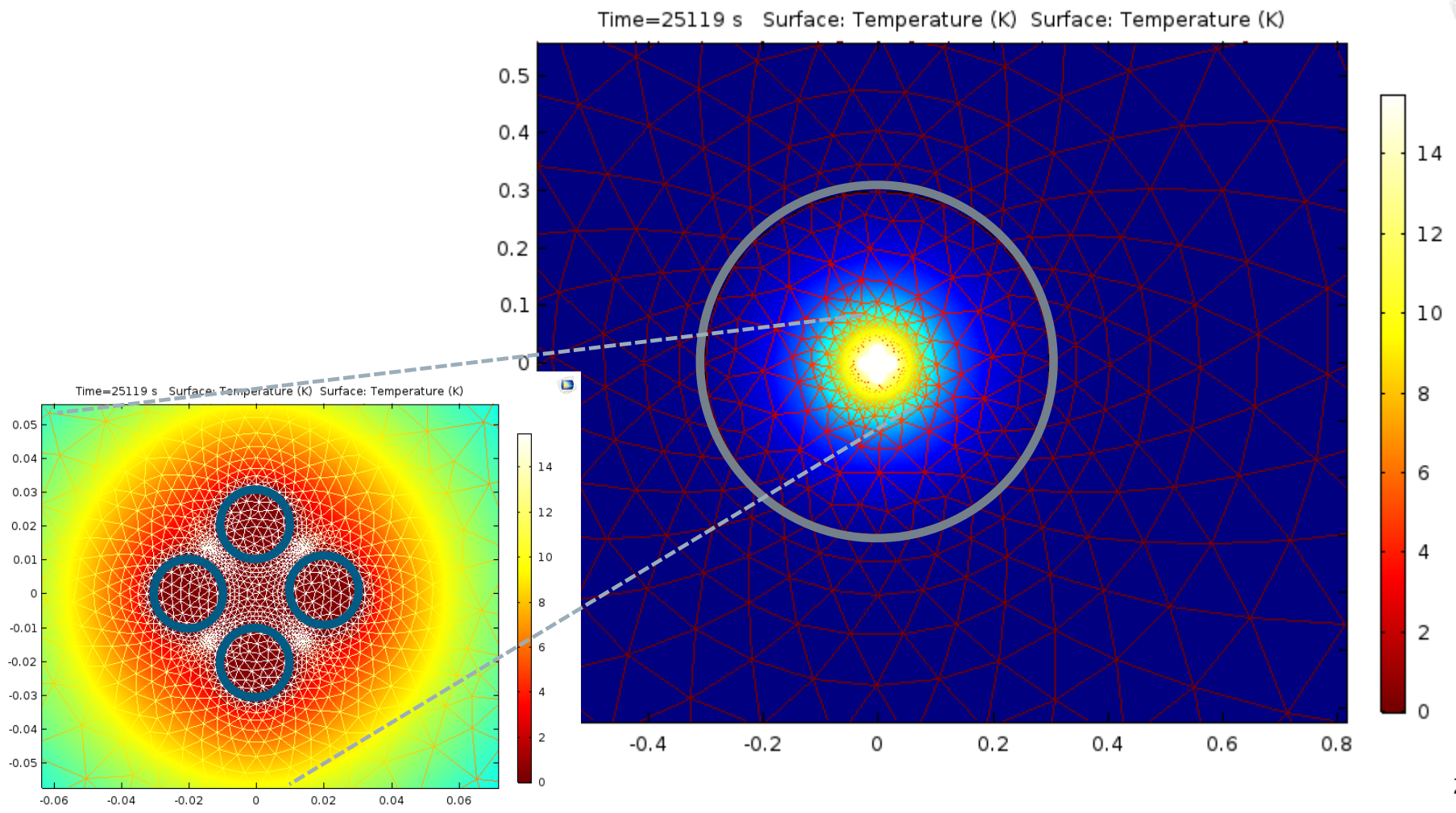
Doublet tracer tests to determine the contaminant flushing properties of a municipal solid waste landfill



N.D. Woodman\*, T.C. Rees-White, R.P. Beaven, A.M. Stringfellow, J.A. Barker

Faculty of Engineering and the Environment, University of Southampton, Southampton SO17 1B, UK.





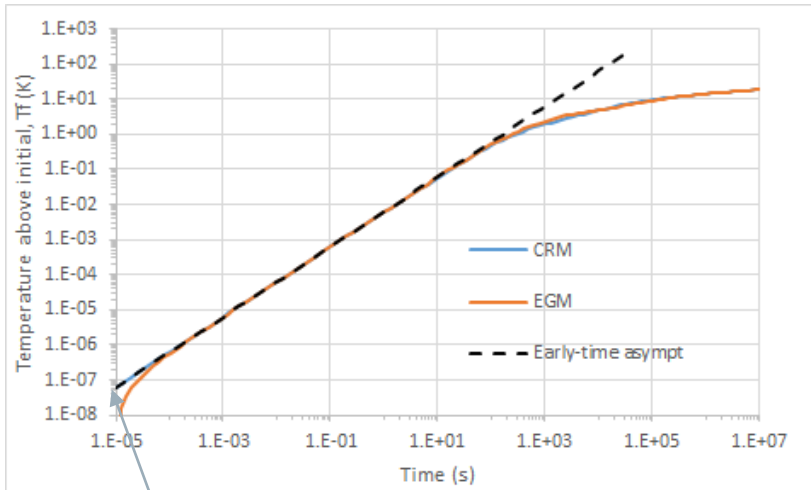
# Simulated Thermal Response Test

- Inject constant heat into the ground (cooling a building) – 50W/m
- Early-time dominated by the fluid thermal capacity
- Late-time dominated by radial heat flow to the ground
- (with correct steady-state pipe and borehole resistances)
- (ignore axial effects which in reality will come into play at later-time)



# Asymptotes

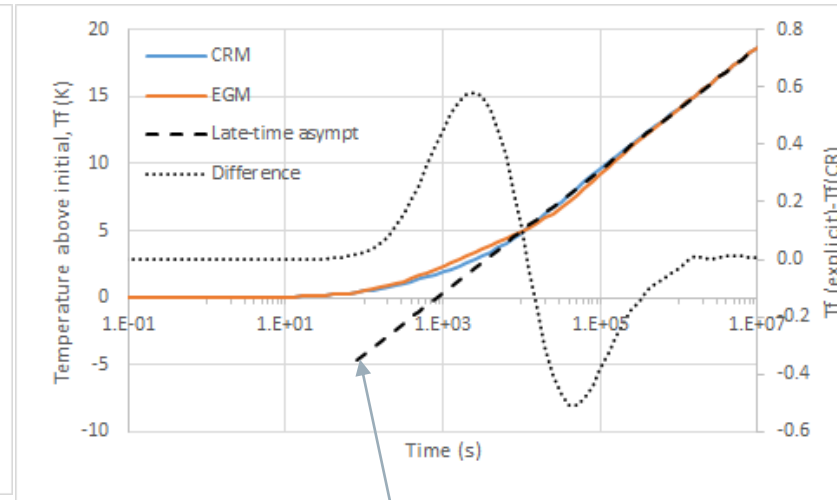
Early-time



$$T_f = qR_p \left( 1 - \exp \left[ -\frac{t}{c_p R_p} \right] \right)$$

$$T_f \sim qt/C_f$$

Late-time ('Jacob approximation')

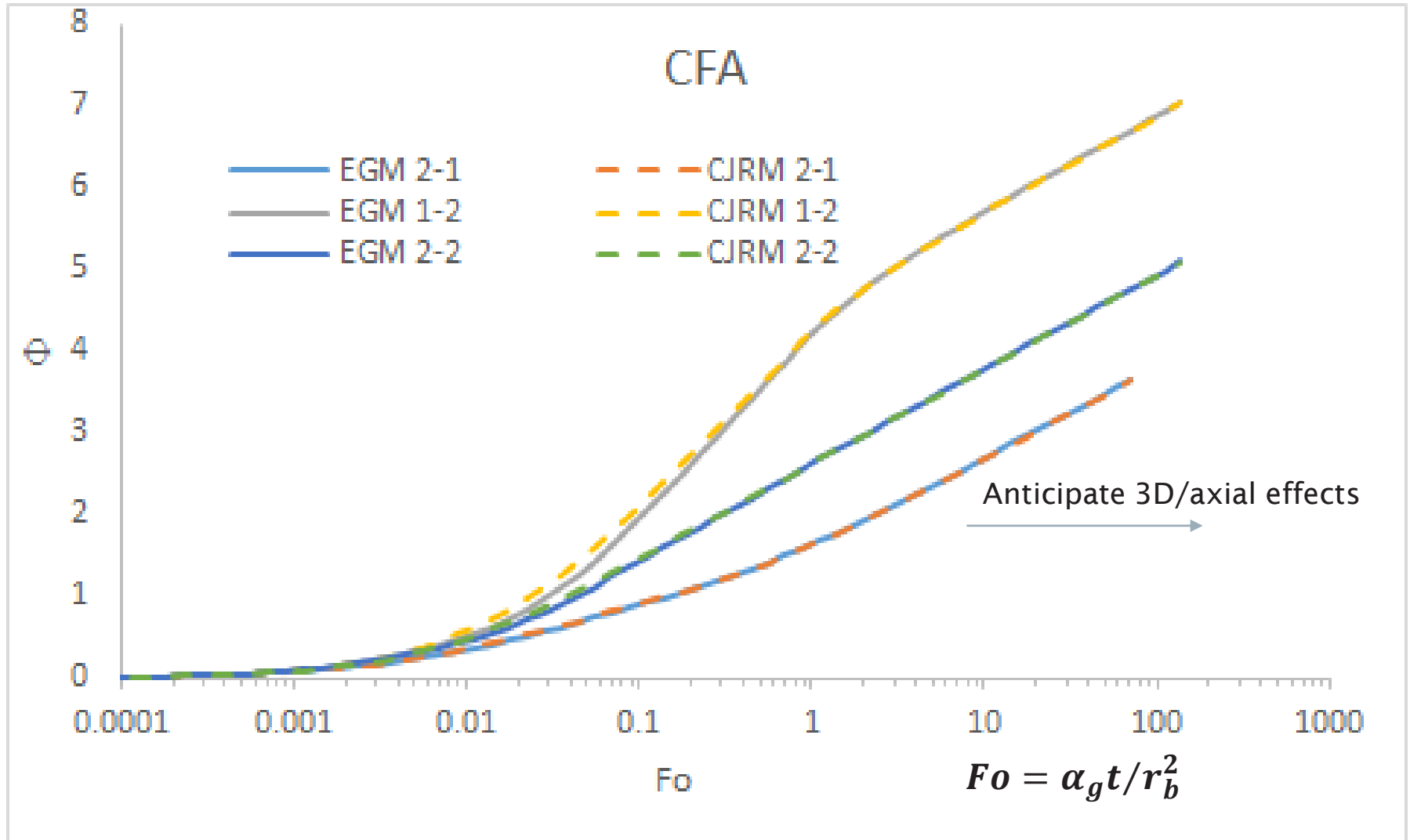


$$T_f \sim \frac{q}{4\pi\lambda_g} \left[ \ln \left( \frac{4\lambda_g t}{c_g \rho_g r_b^2} \right) - 0.5772 \right] + qR_p + qR_b$$

$$Fo = \alpha_g t / r_b^2 > 5$$

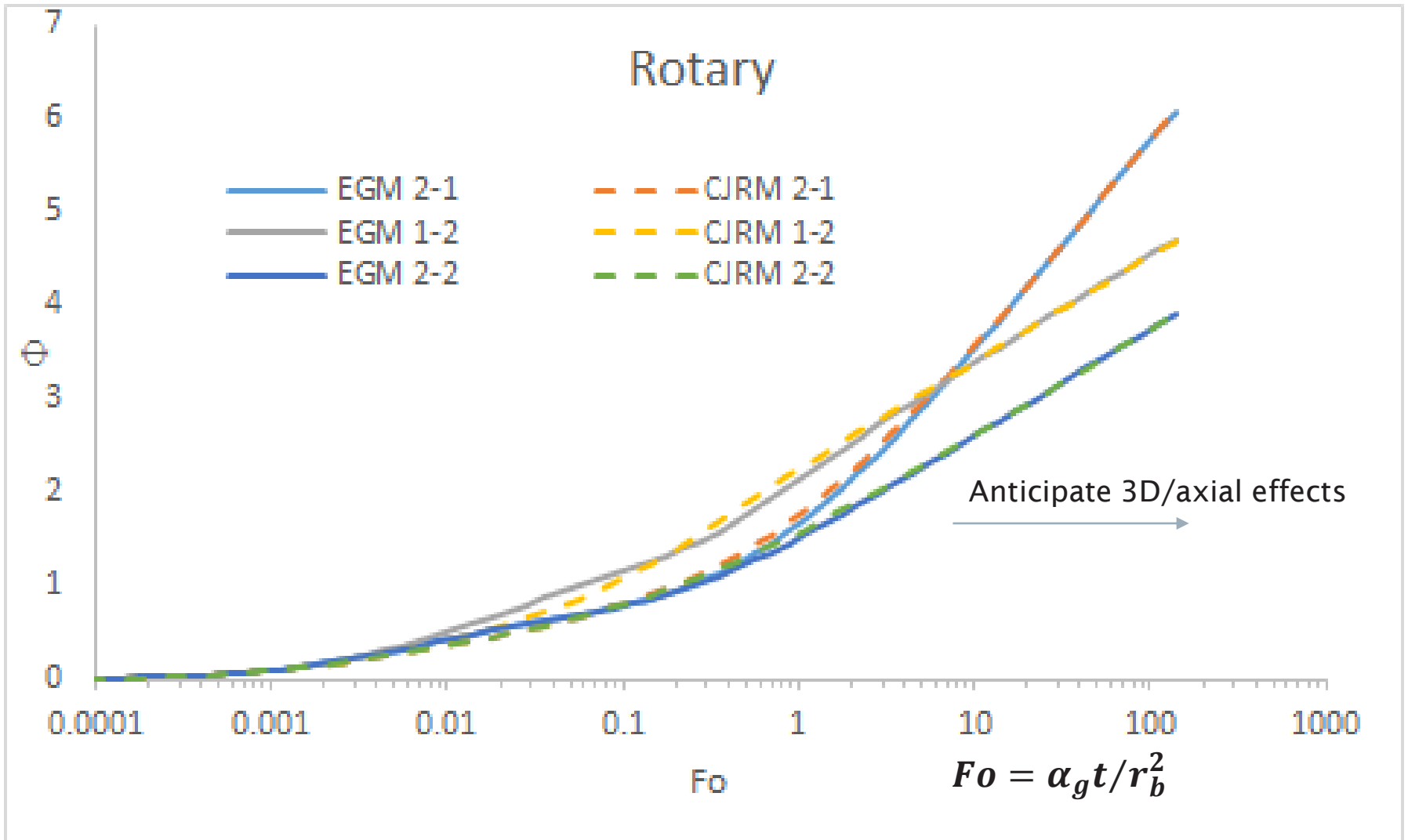
Rotary geometry ( $r_b=300$  mm) for  $(\lambda_c, \lambda_g) = (1, 2)$  W/mK

$$\Phi = 2\pi\lambda_g\Delta T_f/q$$



Maximum fluid temperature discrepancy 0.7°C

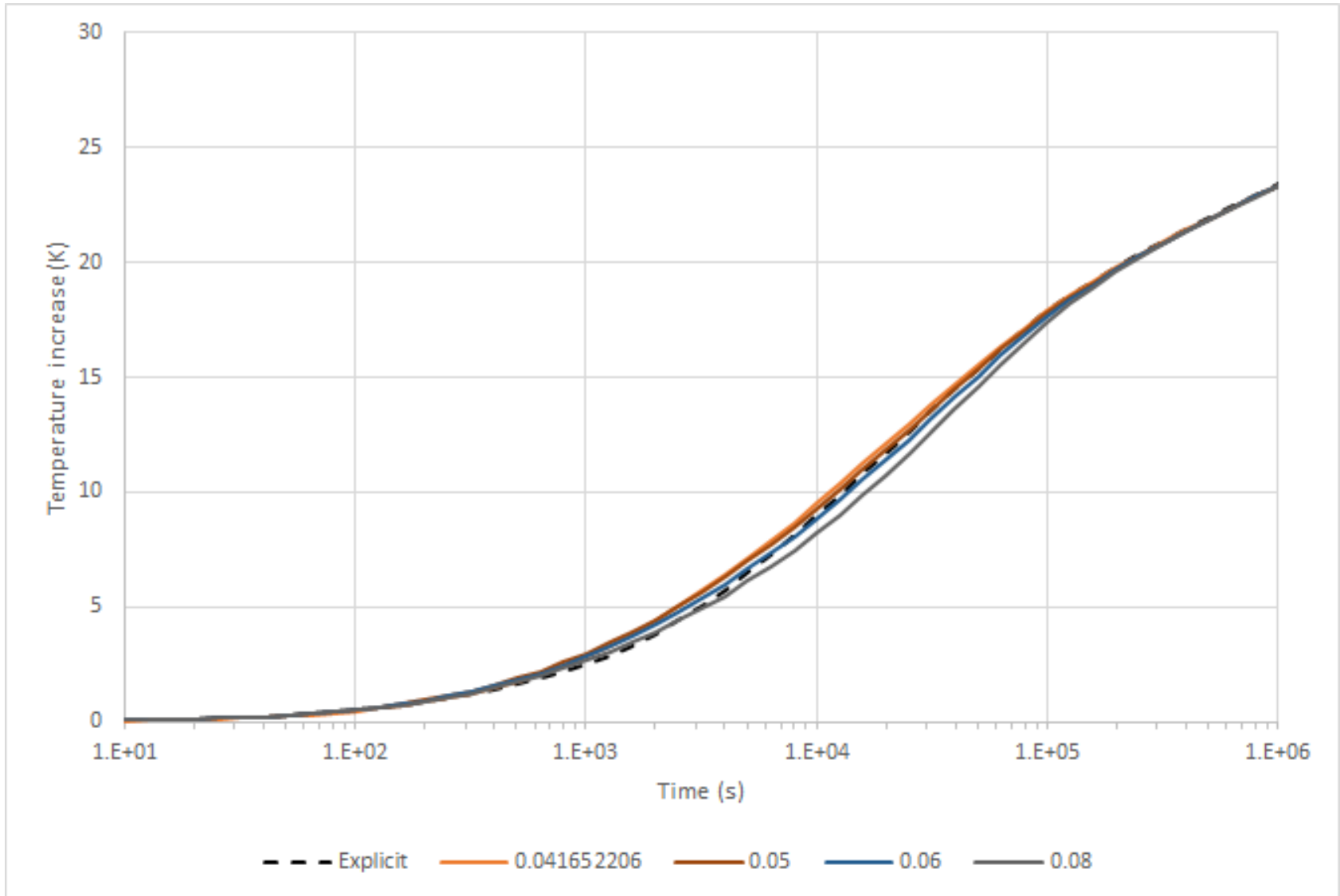
$$\Phi = 2\pi\lambda_g\Delta T_f/q$$



Maximum fluid temperature discrepancy 0.6°C



# Fitting - change $\lambda_c$

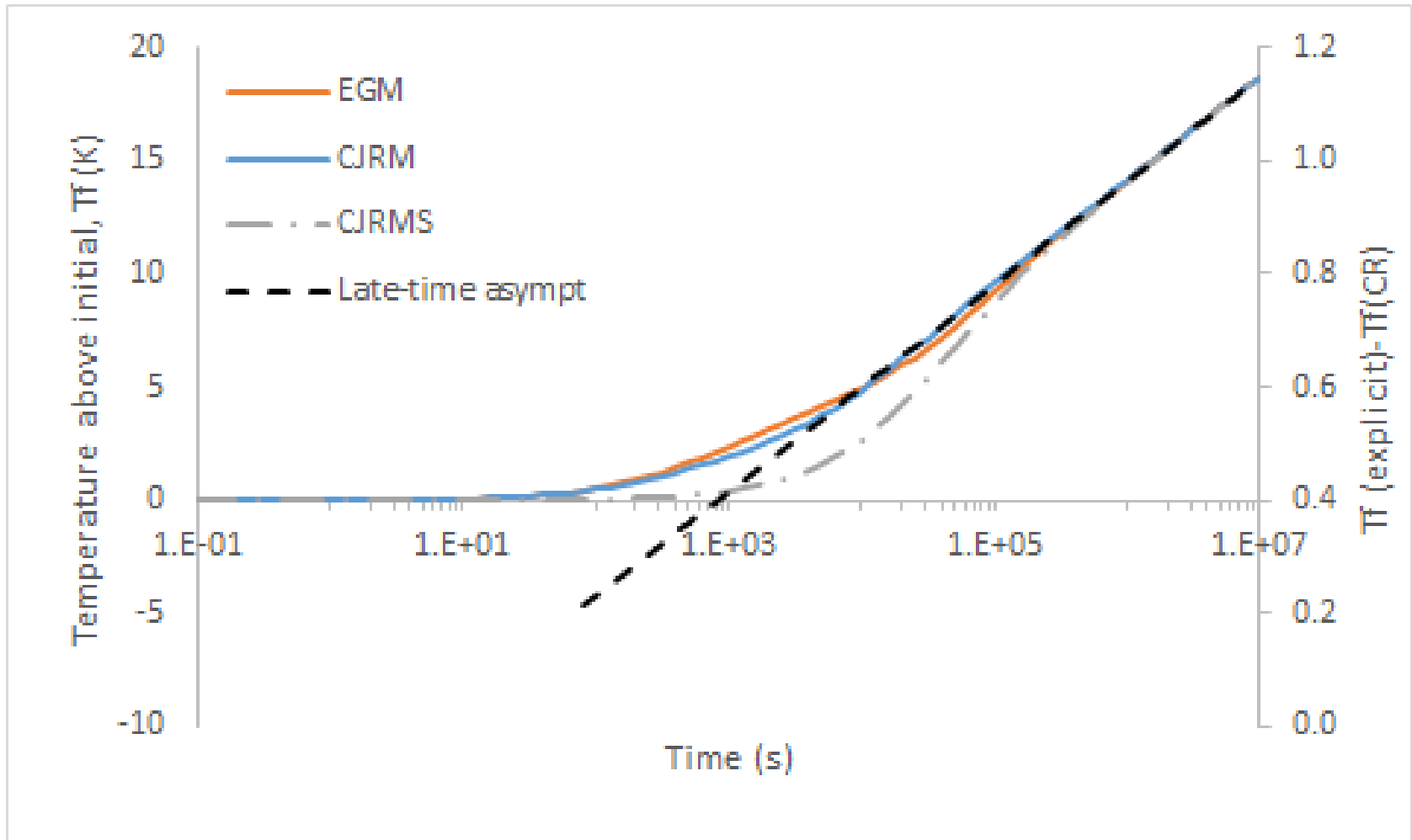


# CJRMS

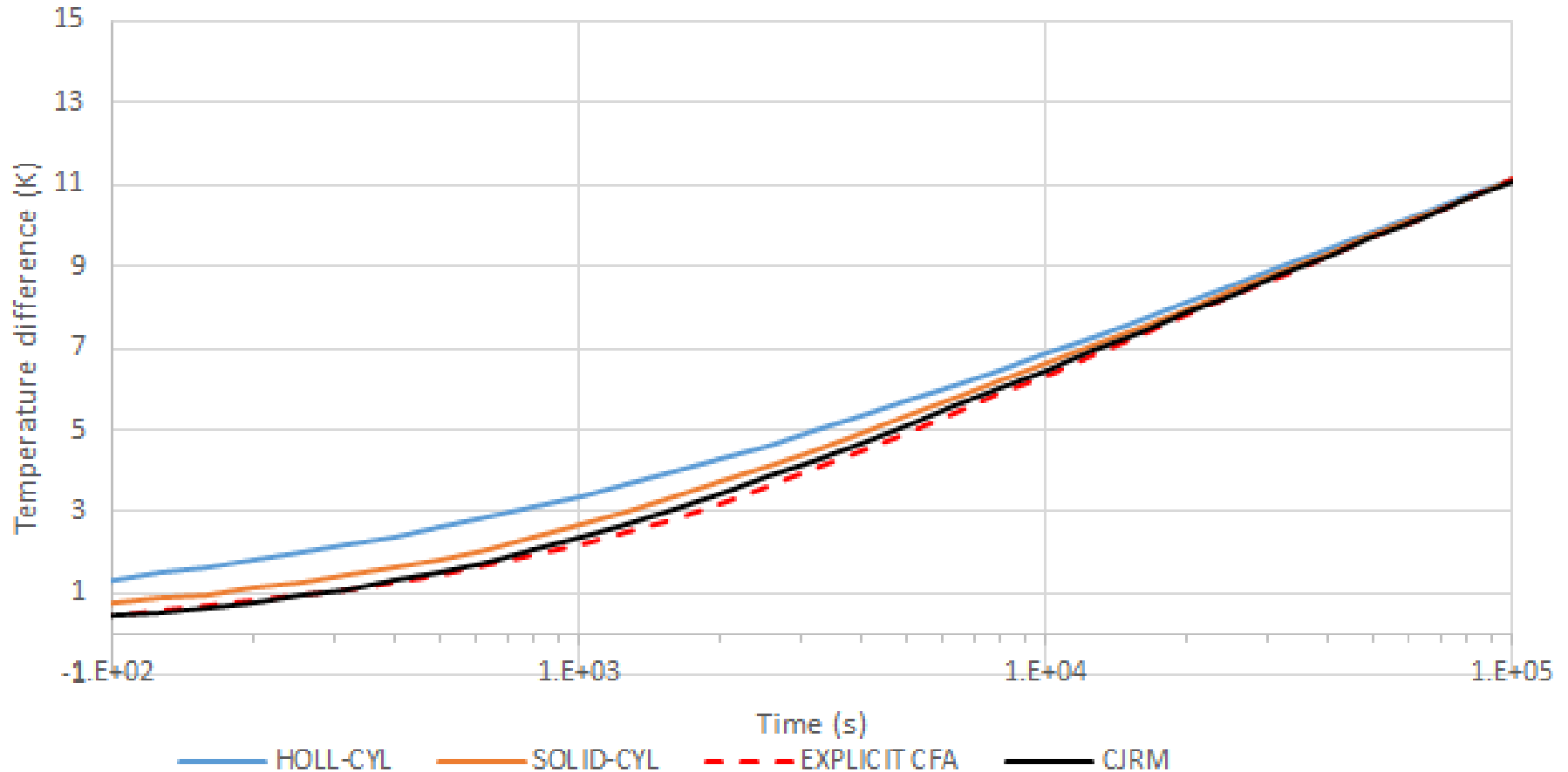
Rotary1-2, 300mm

Makes it worse!

Poor conjecture: not physically realistic



CFA, 300m, 2-2



## Plus points...

- Simple radial model (CJRM) performs well
- Handy addition to the quiver of semi-analytical tools
- CFA and Rotary arrangements matched
- Essentially this because the basic diffusion physics are reproduced, albeit embedding geometrical ‘mistakes’
- (Reinforcing steel makes little difference; not shown here)

## Minus points...

- ‘Mid-time’ error for CJRM
- (although could reduce a little by adjusting  $r_{pe}$  /  $\lambda$ )
- Central store (CJRMS) makes worse fit: reject!
- We are in 2D...need to include axial effects for longer term simulation
- EGMs: smooth numerical error can hide
- So, care with simulating cyclic loads in numerical models...

*We will now simulate a broader range of conditions*

# Acknowledgements



‘Non Steady Analytical Models for Energy Pile Testing and Design’  
(EPSRC [EP/P001351/1](#))